4730 Mechanics 3

1	$0.4(3\cos60^o - 4) = -I\cos\theta$ (= -1) $0.4(3\sin60^o) = I\sin\theta$ (= 1.03920) [$\tan\theta = -1.5\sqrt{3}/(1.5 - 4)$; $I^2 = 0.4^2[(1.5 - 4)^2 + (1.5\sqrt{3})^2]$] θ = 46.1 or I = 1.44 I = 1.44 or θ = 46.1	M1 A1 A1 M1 A1 M1 A1ft [7]	For using I = Δmv in one direction SR: Allow B1 (max 1/3) for $3\cos60^o - 4 = -I\cos\theta$ and $3\sin60^o = I\sin\theta$ For eliminating I or θ (allow following SR case) Allow for θ (only) following SR case. For substituting for θ or for I (allow following SR case) ft incorrect θ or I; allow for θ (only) following SR case.
	Alternatively $I^2 = 1.2^2 + 1.6^2 - 2\times1.2\times1.6\cos60^o$ or $'V'^2 = 3^2 + 4^2 - 2\times3\times4\cos60^o$ I = 1.44 $\frac{\sin\theta}{3(or 1.2)} = \frac{\sin 60}{\sqrt{13(or 2.08)}}$ or $\frac{\sin\alpha}{4(or 1.6)} = \frac{\sin 60}{\sqrt{13(or 2.08)}}$ $and \theta = 120 - \alpha$ θ = 46.1	M1 A1 M1 A1 M1 A1ft A1 [7]	For use of cosine rule For correct use of factor 0.4 (= m) For use of sine rule α must be angle opposite 1.6; (α = 73.9) ft value of I or 'V'
2	2a + 3b = 2×4 b – a = 0.6×4 [2(b - 2.4) + 3b = 8] b = 2.56 v = 2.56	M1 A1 M1 A1 M1 A1 B1ft [7]	For using the principle of conservation of momentum For using NEL For eliminating a ft v = b
3(i)	$2W(a\cos45^o) = T(2a)$ $W = \sqrt{2}\,T$	M1 A1 A1 [3]	For using 'mmt of 2W = mmt of T' AG
(ii)	Components (H, V) of force on BC at B are $H = -T/\sqrt{2}$ and $V = T/\sqrt{2} - 2W$ W(a cosα) + H(2a sinα) = V(2a cosα) [W cosα - T$\sqrt{2}$ sinα = T$\sqrt{2}$ cosα -4Wcosα] T$\sqrt{2}$ sin α = (5W - T$\sqrt{2}$) cos α tan α = 4	B1 M1 A1 M1 A1ft A1 [6]	 For taking moments about C for BC For substituting for H and V and reducing equation to the form X sinα = Y cosα

	Alternatively for part **(ii)** anticlockwise mmt = W(a cosα) +2W(2a cosα + a cos45^o) = T[2a cos(α – 45^o) + 2a] [5W cosα + $\sqrt{2}$ W = T($\sqrt{2}$ cosα + $\sqrt{2}$ sinα) + 2] T$\sqrt{2}$ sin α = (5W - T$\sqrt{2}$) cos α tan α = 4	 M1 A1 A1 M1 A1ft A1 [6]	 For taking moments about C for the whole For reducing equation to the form X sinα = Y cosα
4(i)	[-0.2(v + v^2) = 0.2a] [v dv/dx = -(v + v^2) [1/(1 + v)] dv/dx = -1	M1 M1 A1 [3]	For using Newton's second law For using a = v dv/dx AG
(ii)	 ln (1 + v) = -x (+ C) ln(1+ v) = -x + ln3 [(1 + dx/dt)/3 = e^{-x} ➔ dx/dt = 3e^{-x} -1 ➔e^x dx/dt = 3 – e^x] [-e^x/(3 – e^x)] dx/dt = -1	M1 A1 A1 M1 A1 [5]	For integrating For transposing for v and using v = dx/dt AG
(iii)	[ln(3 – e^x) = -t + ln2] ln(3 – e^x) = -t + ln2 Value of t is 1.96 (or ln{2 ÷ (3 – e)}	M1 A1 A1 [3]	For integrating and using x(0) = 0
5(i)	 Loss of EE = 120(0.5^2 – 0.3^2)/(2×1.6) and gain in PE = 1.5×4 v = 0 at B and loss of EE = gain in PE (= 6) ➔distance AB is 4m	M1 A1 M1 A1 [4]	For using EE = λx^2/2L and PE = Wh For comparing EE loss and PE gain AG
(ii)	[120e/1.6 = 1.5] e = 0.02 Loss of EE = 120(0.5^2 – 0.02^2)/(2×1.6) (or 120(0.3^2 – 0.02^2)/(2×1.6)) Gain in PE = 1.5(2.1- 1.6 – 0.02) (or 1.5(1.9 + 1.6 + 0.02) loss) [KE at max speed = 9.36 – 0.72 (or 3.36 + 5.28)] ½ (1.5/9.8)v^2 = 9.36 – 0.72 Maximum speed is 10.6 ms^{-1}	M1 A1 B1ft B1ft M1 A1 A1 [7]	For using T = mg and T = λx/L ft incorrect e only ft incorrect e only For using KE at max speed = Loss of EE – Gain (or + loss) in PE
	First alternative for **(ii)** x is distance AP [½ (1.5/9.8)v^2 + 1.5x + 120(0.5 – x)^2/3.2 = 120 x0.5^2/3.2] KE and PE terms correct EE terms correct v^2 = 470.4x – 490x^2 [470.4 – 980x = 0] x = 0.48 Maximum speed is 10.6 ms^{-1}	 M1 A1 A1 A1 M1 A1 A1	 For using energy at P = energy at A For attempting to solve dv^2/dx = 0

	Second alternative for **(ii)** [120e/1.6 = 1.5] e = 0.02 [1.5 – 120(0.02 + x)/1.6 = 1.5 $\ddot{x}$ /g] n = $\sqrt{490}$ a = 0.48 Maximum speed is 10.6 ms^{-1}	M1 A1 M1 M1 A1 A1 A1	For using T = mg and T = λx/L For using Newton's second law For obtaining the equation in the form $\ddot{x}$ = $-n^2x$, using (AB – L – e_{equil}) for amplitude and using v_{max} = na.
6(i)	PE gain by P = 0.4g × 0.8 sinθ PE loss by Q = 0.58g × 0.8θ ½ (0.4 + 0.58)v^2 = g × 0.8(0.58θ –0.4sinθ) v^2 = 9.28θ - 6.4sinθ	B1 B1 M1 A1ft A1 [5]	 For using KE gain = PE loss AEF
(ii)	 0.4g sinθ – R = 0.4v^2/0.8 [0.4g sinθ – R = 4.64θ – 3.2 sinθ] R = 7.12 sinθ – 4.64θ	M1 A1 M1 A1 [4]	For applying Newton's second law to P and using a = v^2/r For substituting for v^2 AG
(iii)	R(1.53) = 0.01(48...), R(1.54) = -0.02(9...) or simply R(1.53) > 0 and R(1.54) < 0 R(1.53) × R(1.54) < 0 ➔1.53 < α < 1.54	M1 A1 M1 A1 [4]	For substituting 1.53 and 1.54 into R(θ) For using the idea that if R(1.53) and R(1.54) are of opposite signs then R is zero (and thus P leaves the surface) for some value of θ between 1.53 and 1.54. AG
7(i)	T_{AP} = 19.6e/1.6 and T_{BP} = 19.6(1.6-e)/1.6 0.5g sin30° + 12.25(1.6 – e) = 12.25e Distance AP is 2.5m	M1 A1 M1 A1ft A1 [5]	For using T = λe/L For resolving forces parallel to the plane
(ii)	Extensions of AP and BP are 0.9 + x and 0.7 – x respectively 0.5g sin 30° + 19.6(0.7 – x)/1.6 – 19.6(0.9 + x)/1.6 = 0.5 $\ddot{x}$ $\ddot{x}$ = -49x Period is 0.898 s	B1 B1ft B1 M1 A1 [5]	 AG For stating k < 0 and using T = 2π/$\sqrt{-k}$
(iii)	 $2.8^2 = 49(0.5^2 – x^2)$ x^2 = 0.09 x = 0.3 and -0.3	M1 A1ft A1 A1ft [4]	For using $v^2 = \omega^2(A^2 – x^2)$ where ω^2 = -k ft incorrect value of k May be implied by a value of x ft incorrect value of k or incorrect value of x^2 (stated)